# Spin an M&M™ on a hard, level surface and it will rise up on its edge.

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**10 March 2009**

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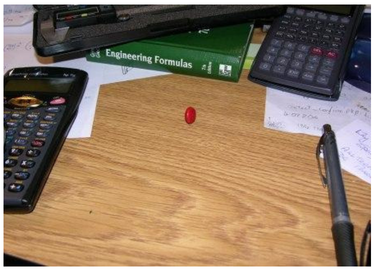
(2)



(3)

(1) A stationary M&M™ lies flat. (2) When given a brisk spin, it stands up and spins on its edge. (3) Using a 1/60th of a second exposure, the M&M™ appears to stand on its edge.

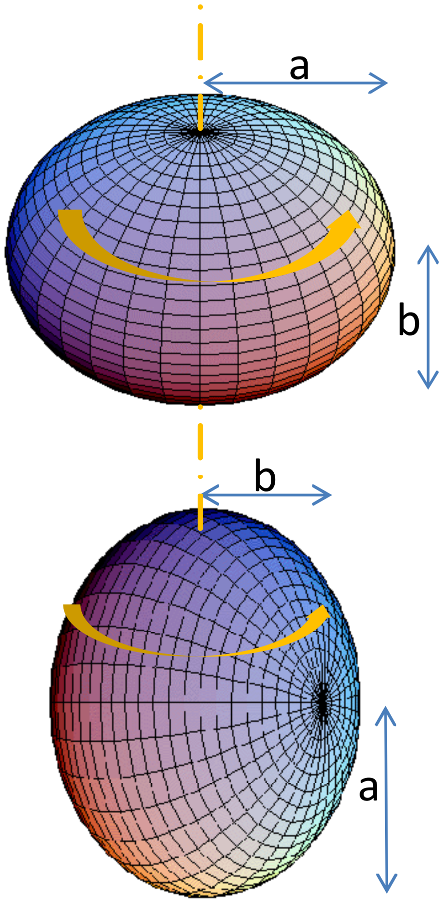
## Does this not violate the law of gravity?

Hold on there – the law of gravity does not say that things can *never* go up; only that what goes up must come down. Stated more exactly, things can only go "up" or against gravitational pull for as long as they have enough energy to do so.

## Where does the energy come from that causes the M&M™ to stand up, and why does this happen anyway?

An unrestrained rotating body seeks to rotate about the axis for which its rotational energy is a minimum while having the same angular momentum. Rotation about an axis that does not meet these criteria is unstable and tends to start wobbling. When the minimum-energy axis of rotation is found in the midst of all this wobbling, the rotation becomes stable about that new axis.

In the case of an M&M,™ by pure fluke of geometry, rotation about the long axis requires less energy than about the short axis. When you spin it flat, it tries to stand up because spinning on its edge is a more stable mode of rotation. The energy required to lift it up against gravity comes from the difference in energy between the unstable flat spin and the more stable edge spin. On the following pages we will work out how fast the M&M™ must spin in order to perform this nifty trick.




At rest, the M&M™ (modelled with only slight inaccuracy as an ***oblate spheroid*** or squashed ball) normally lies flat. When up on its edge, its centre of mass is higher by an amount equal to the difference between the major axis ***a*** and the minor axis ***b***. We'll call the mass ***m***, and ***g*** of course is the gravitational acceleration at the earth's surface, 9.8 m/s². The difference in gravitational potential energy is therefore

$$mg(a-b) \qquad (1)$$

For a plain M&M™ with average mass 0.936 ± 0.123 grams,* major axis 6.83 ± 0.17 mm* and minor axis 3.65 ± 0.30 mm,* this energy threshold is found to be 28 μJ (28 millionths of a Joule). If at least this much energy would be left over if the axes of rotation were to be reversed, then the M&M™ will swap axes, stand up and spin on its edge. The only constraint is that the angular momentum must be the same.

The simplest analysis is done by taking a before and after snapshot, and ignoring the complicated wobbling that occurs during the transition. We assume an initial angular velocity $\omega_1$ about the short axis, for which the momentum is $I_1\omega_1$. (The letter ***I*** represents the "moment of inertia" or the resistance of the body to rotations about a particular axis.) The final angular speed is $\omega_2$ about the long axis with final momentum $I_2\omega_2$. We wish to find the minimum initial angular speed $\omega_1$ that will permit the M&M™ to stand up and spin about its long axis.

The initial rotational energy must be more than the final energy by at least the amount required to lift the M&M™ onto its edge. Therefore,

$$\begin{aligned} E_1 &\geq E_2 + mg(a-b) \\ \tfrac{1}{2} I_1 \omega_1^2 &\geq \tfrac{1}{2} I_2 \omega_2^2 + mg(a-b) \end{aligned} \qquad (2)$$

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* 95% of the M&Ms™ I measured fell within this range. Unfortunately I cannot prove this for a fact because I later ate all the specimens. Occasionally an M&M is found that ***does not work***, or that requires speeds higher than are easy to produce.

 

The other piece of physics we need now is the requirement that angular momentum stays pretty much constant. Over short periods of time and if friction is low, this will be true:

$$I_1 \omega_1 = I_2 \omega_2 \qquad 3$$

All we need now are expressions for $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$, the moments of inertia of an oblate spheroid. Finding things like this is not as hard as it sounds if you a) know the name of it, b) have access to the internet, and c) have enough experience to recognize the correct answer when you see it. And here they are:

$$I_1 = \tfrac{2}{5} m a^2 \qquad\qquad I_2 = \tfrac{1}{5} m (a^2 + b^2) \qquad 4$$

After working it all out and solving for $\boldsymbol{\omega_1}$, I found the following:

$$\omega_1 = \sqrt{\frac{10 g (a+b)}{a^2 - b^2}} \qquad 5$$

(As often happens in these kinds of analyses, the mass has dropped out completely, leaving only geometry and gravity.)

With the measurements I took, the initial required speed works out to about 97 radians per second (say what?) or in everyday terms, about 930 revolutions per minute (RPM). The final speed is higher, at about 1430 RPM. That feels about right to me, though I have not measured the speed myself. If you figure out how to do it using items you already have at your desk, please let me know! Maybe a laser pointer, the IR port of your i-phone, who knows. Get creative.

Actually, there is freeware available that will turn your computer's sound card and microphone into a cheapo spectrum analyzer. By "listening" to the sound of the spinning M&M™ you may be able determine it's speed using a bit of luck and some educated guesses. The primary frequency of 15 – 20 Hz is likely too low to measure directly, so you'd have to be watching the harmonics. Other freeware makes your PC speaker into a programmable function generator. Perhaps connecting the output to an LED driver can make a simple strobe light. When the spinning M&M™ appears stationary, the speed of rotation is either half, double, or the same as the strobe frequency.

One more thing. We glossed over the important point of how exactly the transition from spinning flat to standing up occurs. When an unstable rotation has a slight wobble in it, that wobble does not correct itself. Instead, it grows exponentially until the axis of rotation changes completely.

If the M&M™ were a perfectly shaped symmetrical object, the transition from an unstable rotation into a stable one might not happen at all unless you give it a bit of a kick to start it off. But because an M&M™ is always slightly irregular in shape, it always wobbles a bit anyway and the transition happens spontaneously and reliably.

What if the rotational energy difference is much larger than that needed to lift the M&M™ up onto its edge? It appears that the equations are favouring one particular speed of rotation at which this happens. Since there would be nowhere else for any additional energy to go, an unstable orbit would most likely be forced to continue until friction, noise and air resistance removes the excess energy and the optimum speed is reached. It would therefore be very interesting to discover whether an M&M™ always stands up at a certain speed, regardless of how fast we spin it to begin with.

Another interesting experiment would be to spin the M&M™ on an ice block or other low-friction surface. Without friction, would it still work? (Theory says no: it needs something to torque against.)

One practical application of this concept is the stability of computer hard disk drives. How thick can the complete disk assembly be made before the rotation becomes unstable and bearing life is reduced? Does lightening the disk near the axis improve stability?

Another application is satellite attitude control. Some satellites rotate in orbit, and if the design can't accommodate rotation around the stable axis, then other means of controlling the spin must be applied. These problems have of course all been solved, though it is interesting to me that 300-year-old science is still necessary for modern technology to work.

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M&M is a trademark of the Mars corporation, who neither supported nor endorsed this research. I'm sure they don't mind, though. I'm one of their best customers.