A HYDROSTATIC BEARING WITH COMPRESSIBLE FLUID FOR BROAD APPLICATION

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ABSTRACT
The use of relatively inviscid, compressible fluids in externally-pressurized bearings has interesting possibilities for both OEM and retrofit applications. The chance to dramatically reduce mechanical losses and bearing heating, the elimination of oil from the process and installation, and the utilization of compressible process fluids as the supporting medium all have potential economic and environmental benefits. An experimental gas bearing rig was constructed to investigate the feasibility of some general applications. Clearance and orifice dimensions were selected based on a fairly simple gas flow model. Bently-Muszynska model parameters for the hydrostatic gas bearing were obtained through static-pull and non-synchronous perturbation testing.

INTRODUCTION
The present work describes efforts to identify and interpret the Bently-Muszynska model parameters of a hydrostatic gas bearing. A great deal of work has been done to analyze externally-pressurized compressible-fluid bearings (for a sample see [1-3]) but an experimental description of a gas bearing using the Bently-Muszynska model has not been reported. Previous work at this institution [4] focused on hydrostatic oil bearings assisted by an active positional feedback loop. The purpose of this was to optimize a rotor-bearing model through selection of bearing characteristics, then to build an externally-pressurized bearing that reproduces those optimal characteristics. It was found that those ends could be met in many cases by replacing the active elements with passive compensation through appropriately-sized orifice restrictions in the pocket supply lines (by no means a new approach). The original intent has been retained, and this design process must now be extended to non-traditional bearing fluids. The present work is an attempt to establish an initial, experimental relationship between a specific gas bearing geometry and its Bently-Muszynska model parameters.

NOMENCLATURE
A = rotor response amplitude
C_d = discharge coefficient
C_d0 = orifice discharge coefficient
D = damping coefficient
DDS = direct dynamic stiffness
D.S. = dynamic stiffness
F = magnitude of radial forces
K = modal or “spring” stiffness
M = modal rotor mass
P = pressure at entrance to bearing clearance
P_a = ambient pressure
P_s = supply pressure
QDS = quadrature dynamic stiffness
R = specific gas constant
R_c = critical pressure ratio for sonic flow
T = absolute temperature
a = orifice cross-sectional area
h = radial clearance at a given bearing segment
j = \sqrt{-1}
k = gas specific heats ratio
l = bearing half-length
r = rotor response vector
r_b = bearing radius
w = mass flow rate through orifice
Ω = shaft rotative speed
α = response phase
δ = phase or orientation of radial forces
λ = fluid circumferential average velocity ratio
μ = dynamic viscosity
ω = frequency of applied rotating radial force
DESCRIPTION OF TEST APPARATUS

A test stand was constructed to allow determination of the bearing stiffness and damping independently of any rotor characteristics. The rotor was designed to operate in a rigid-body conical mode, with the test bearing located at the antinode.

The rotor has a total length of 460 mm (18 in.) and is 50.8 mm (2 in.) in diameter. The drive end is supported in a precision rolling-element bearing, while the non-drive end is supported in the test bearing. The midplanes of the bearings are 305 mm (12 in.) apart. A balance wheel or a non-synchronous perturbation wheel can be attached to the outboard non-drive end of the rotor. Eddy-current displacement transducers are mounted in X-Y configuration on the inboard and outboard ends of the test bearing. Using the conical mode shape, displacements and applied forces can be projected into the test bearing midplane, enabling the use of a simple, single-plane rotor model.

The test bearing consists of a carbon-graphite cylindrical sleeve, in which radial holes have been machined. Orifice inserts were made which fit into these holes and produce the appropriate level of hydrostatic compensation. The orifice inserts produce a recess in the bearing wall at most 0.13 mm deep at the center by 7.92 mm in diameter, minimized to avoid pneumatic instability [5]. The carbon-graphite wall provides for fail-safe operation in the event of a supply pressure loss.

Compressed air was used as the supporting medium of the test bearing. A shutoff valve, regulator and dryer/filter were used to deliver air into the bearing housing. A recessed groove in the housing bore encircling the sleeve distributes pressure evenly to the four orifice inserts. The supply pressure, \( P_0 \), is assumed to be the pressure within this groove, and is measured by a gauge mounted just outside the housing. Air passes from there through the orifice restrictions and is introduced into the bearing clearance. The clearance exhausts directly to ambient conditions.

DESCRIPTION OF TESTS

To measure characteristics of the test bearing, two types of tests were performed: static pull tests and non-synchronous sweep perturbation with a known unbalance.

Static pull tests were performed by attaching a force gauge to the rotor through a free-spinning disk. This allowed force-displacement measurements to be taken with the rotor at speed. The applied force is varied using a threaded turnbuckle. Static pull tests were performed at various supply pressures and various rotor speeds. Rotor response was measured as displacement in two dimensions to determine both the magnitude and angle of the response.

Non-synchronous perturbation tests were performed by attaching a free-spinning disk to the end of the rotor and placing calibrated unbalance weights at a known radius on the disk. The disk was driven by a DC motor independently of the shaft rotative speed. Rotor displacement waveforms were filtered to the forcing frequency (equal to the disk rotative speed). A once-per-turn marker (Keyphasor™) with a known angular relation to the unbalance weight was used to observe the perturbator drive and establish a reference for phase measurements. The rotor speed was held constant while sweeping the perturbation speed. For each set of conditions, two tests were performed, with the unbalance weight moved 180° between tests. Rotor response data (amplitude and phase) from successive tests was subtracted vectorially to eliminate unknown residual unbalances, runout and other constant sources of error, and to increase the sensitivity of the test.

BENTLY-MUSZYNSKA MODEL

Data from the test stand was analyzed using the symmetric, single-plane reduction of the Bently-Muszynska rotor model [6], and the dynamic stiffness approach. The simplified model is

\[
M\ddot{r} + D\dot{r} + (K - jD\lambda\Omega)r = F e^{j(\alpha + \delta)}. \tag{1}
\]

Rotor response is assumed to have the form

\[
r = Ae^{j(\alpha + \beta)}, \tag{2}
\]

noting that the frequency of the linear response is equal to the frequency \( \omega \) of the forcing function. This leads to the solution

\[
Ae^{j\alpha} = \frac{Fe^{j\delta}}{K - M\omega^2 + jD(\omega - \lambda\Omega)}. \tag{3}
\]

Dynamic stiffness represents rotor-bearing characteristics \( K, M, D \) and \( \lambda \). Since the conical mode shape dominates at the speeds tested, the rotor contributions to \( K \) and \( D \) are assumed negligible. To determine these parameters experimentally, one divides the known applied force by the measured response to obtain the complex dynamic stiffness:
\[ D.S. = DDS + jQDS = \frac{F_e e^{j\delta}}{A e^{j\alpha}} = K - M\omega^2 + jD(\omega - \lambda\Omega) \quad (4) \]

The real and quadrature components are termed the direct dynamic stiffness (DDS) and the quadrature dynamic stiffness (QDS), respectively. The model coefficients are assumed to be constant. If one expects coefficients to vary with large changes in either \( F \) or \( A \), then the “local” or differential dynamic stiffness may be determined from the finite vector differences:

\[
D.S._{\text{Local}} = \frac{\Delta(F e^{j\delta})}{\Delta(A e^{j\alpha})}.
\quad (5)
\]

Static pull tests apply a non-rotating force to the rotor, making \( \omega = 0 \). If the shaft rotative speed, \( \Omega \), is also zero, then the result is a direct measurement of the modal bearing stiffness \( K \).

\[
D.S._{\text{STATIC PULL}} = K.
\quad (6)
\]

Direct stiffness exists in a hydrostatic bearing even when the journal is not rotating. \( K \) is therefore assumed independent of \( \Omega \), though it may be a nonlinear function of eccentricity. If the shaft is allowed to rotate, direct dynamic stiffness acquires a quadrature component:

\[
D.S._{\text{STATIC PULL}} = K - jD\lambda\Omega.
\quad (7)
\]

The forced rotor position assumes an attitude angle defined as

\[
\alpha - \delta = a \tan \left( -\frac{D\lambda\Omega}{K} \right).
\quad (8)
\]

This is understood to be the result of hydrodynamic wedge formation, and is also used to predict whirl instability threshold. One expects \( \lambda \), the fluid circumferential average velocity ratio, to be essentially zero for a purely hydrostatic bearing, and approximately 0.47 to 0.5 for a typical hydrodynamic bearing.

Non-synchronous perturbation provides the most general form of the model,

\[
D.S. = K - M\omega^2 + jD(\omega - \lambda\Omega).
\quad (9)
\]

Only this form permits a full determination of each of the parameters. However, static pull tests are useful for determining local variations of some parameters with eccentricity ratio.

### Compressible Fluid Flow Through a Hydrostatic Bearing

The mass flow rate through an orifice restriction [7] is described as

\[
w = \begin{cases} 
C_{d,a}P \left[ \frac{2k}{(k-1)RT} \left( \frac{P}{P_i} \right)^{\frac{k-1}{k}} - \left( \frac{P}{P_i} \right) \right] & \text{for } \frac{P}{P_i} > R_e \\
C_{d,a}P \left[ \frac{2k}{(k-1)RT} \left( R_i^2 - (R_i)^{\frac{k}{k-1}} \right) \right] & \text{for } \frac{P}{P_i} \leq R_e 
\end{cases}
\quad (10)
\]

There are two flow regimes represented. The first, valid for \( P \) above the critical pressure, is characterized by a dependence of the flow rate on the downstream clearance pressure \( P \). In the second regime, \( w \) is independent of \( P \) and represents sonic or “choked” flow condition through the orifice, in which any changes in downstream conditions do not affect upstream conditions. A choked orifice behaves as a constant mass flow regulating device, which is highly advantageous for hydrostatic bearing operation. See [8] for an explanation of how hydrostatic film stiffness is greater and pumping losses lower when the feed rate per pocket is held constant.

For simplicity, the flow through the annular clearance of the bearing is treated as one-dimensional flow through a slot [9]. Since there are four feedports, the annulus is divided into four sectors, each with an assumed constant gap, a length of one-half the bearing length, and a width of one-fourth the bearing circumference. The journal is assumed to be at equilibrium between one pair of opposing feedports so they may be ignored, and to have a specified eccentricity between the other pair. One-half the flow through each orifice is assumed to pass through a corresponding slot. The mass flow equation is therefore

\[
w = \frac{C_{d,a} \pi r_i h^3 (P^2 - P_a^2)}{24 \mu RTl}.
\quad (11)
\]

These two equations with two unknowns (\( P \) and \( w \)) are solved numerically for two opposing feedports. This is done as follows: a value for \( P \) is assumed and used to compute \( w \) from Eq. 10. Then, Eq. 11 is used to compute \( P \) again from \( w \). A new value for \( P \) is obtained by integrating the resulting error, and the process is repeated. Linearization through the \( \tanh \) function is used to keep \( P \) bounded by the supply and atmospheric pressures. Convergence to a tolerance of \( 10^{-3} \) usually takes less than five iterations. The difference between the results for opposing feedports is multiplied by an effective area (obtained by assuming a triangular pressure profile across the lands) to obtain the net radial force. The resulting force is an estimation of the radial load required to displace the rotor into the assumed...
eccentric position. The solution is repeated for a range of eccentricities to obtain the distribution of load over the eccentricity.

It was found that total load capacity predicted by this model is primarily a function of supply pressure, and largely independent of radial clearance or orifice diameter. Average stiffness likewise is largely controlled by the maximum load divided by the radial clearance. However, the distribution of load as a function of eccentricity could be affected to produce local regions of higher or lower than average stiffness. The orifice diameter primarily affects this distribution.

EXPERIMENTAL RESULTS

Figure 2 shows predictions made using Eqs. 10 and 11 with parameters from the test bearing. Figure 3 is the corresponding plot of experimental data from the test rig for $\Omega = 0$ RPM. Load was determined within 1 lb (4.4 N), and eccentricity was determined within about 15% error. The results are largely consistent with the predictions, even given the simplicity of the model, but with some notable exceptions. Increasing pressure did not result in observed proportional increases in maximum load as predicted. Also, at higher pressure, the distribution of load differs significantly from predictions. This can be attributed to the effects of supersonic shock boundary formation around the orifice outlet which reduces the effective area over which the assumed static pressure can act [10]. In addition, a single discharge coefficient could not be found which reproduced the experimental flow rates for all pressures, contributing to further discrepancy.

Table 1 shows the average direct dynamic stiffness for the same tests. A distinct dependence on eccentricity was noted, so the average results are reported for eccentricity ratios less than 0.5 and greater or equal to 0.5 separately. Quadrature stiffness depends strongly on an accurate measurement of the attitude angle (the relative angle between applied force and displacement). Since angular errors of about 3 degrees were common, the quadrature stiffness was assumed to have errors of about 5% of the overall stiffness magnitude for low attitude angles, since $\sin(3^\circ) = 0.05$. These data show that in several cases there are significant differences in quadrature stiffness between low and high eccentricity ratios.

<table>
<thead>
<tr>
<th>$P_s$ (psig)</th>
<th>0 RPM</th>
<th>5 kRPM</th>
<th>10 kRPM</th>
</tr>
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<tbody>
<tr>
<td>14</td>
<td>21</td>
<td>24</td>
<td>29</td>
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<td>25</td>
<td>51</td>
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<td>100</td>
<td>152</td>
<td>139</td>
<td>138</td>
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</table>

<table>
<thead>
<tr>
<th>$P_s$ (psig)</th>
<th>0 RPM ecc. &lt; 0.5</th>
<th>5 kRPM ecc. &lt; 0.5</th>
<th>10 kRPM ecc. &lt; 0.5</th>
<th>0 RPM ecc. &gt; 0.5</th>
<th>5 kRPM ecc. &gt; 0.5</th>
<th>10 kRPM ecc. &gt; 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.02 ±1</td>
<td>-5.8 ±1.3</td>
<td>-8.5 ±1.3</td>
<td>-17 ±2</td>
<td>-19 ±2</td>
<td>-12 ±2</td>
</tr>
<tr>
<td>25</td>
<td>-2.5 ±2.6</td>
<td>-4.8 ±2.3</td>
<td>-8.9 ±2.3</td>
<td>-14 ±2.5</td>
<td>-20 ±2.5</td>
<td>-12 ±4.5</td>
</tr>
<tr>
<td>50</td>
<td>-0.1 ±4.9</td>
<td>-8 ±4.6</td>
<td>0.56 ±4.6</td>
<td>-2.8 ±4.5</td>
<td>-12 ±4.5</td>
<td>-2.8 ±4.5</td>
</tr>
<tr>
<td>75</td>
<td>-1.4 ±6.5</td>
<td>-6.0 ±6.0</td>
<td>-12 ±6.0</td>
<td>-0.04 ±6.1</td>
<td>-9.1 ±6.1</td>
<td>-9.1 ±6.1</td>
</tr>
<tr>
<td>100</td>
<td>-1.2 ±7.6</td>
<td>-1.9 ±7.0</td>
<td>-10 ±7.0</td>
<td>-0.42 ±6.9</td>
<td>-2.9 ±6.9</td>
<td>-2.9 ±6.9</td>
</tr>
</tbody>
</table>
Figure 4 shows the quadrature dynamic stiffness plots for 25 psig (172 kPa) supply pressure as typical. The zero-cpm intercept correlates well to the static pull tests. The linear curve fit leads to the identification of the parameters \( D \) and \( \lambda \) (see Eq. 9). The zero-cpm intercept is equal to \(-D\lambda\Omega\) (same as static pull QDS), and the slope of the line is \( D \). These parameters are summarized in Table 3. In contrast to the static pull results, these values represent averages over a range of dynamic eccentricity ratios rather than values at specific eccentricities. Thus, the values do not necessarily correlate to either the high or low eccentricity values in Table 2, but to their average instead.

Not shown are the values of direct dynamic stiffness computed from non-synchronous tests and extrapolated down to zero cpm. These values correspond predictably and unremarkably to those determined through static pull tests.

| Table 3. Summary of QDS parameters from non-synchronous tests. |
|-----------------------------|-----------------------------|-----------------------------|
| \( \Omega \)               | \( \Omega \)               |
| 25 psig                     | 50 psig                     | 100 psig                    |
| \( D \) \( \lambda \)      | \( D \) \( \lambda \)      | \( D \) \( \lambda \)      |
| 0                           | 32                          | -                           |
| 25                          | 25                          | 20                          |
| 5,000                       | 36                          | 0.54                        |
| 10,000                      | 36                          | 0.50                        |

DISCUSSION OF RESULTS

The bearing characteristics investigated are seen to vary in a predictable manner within the parameters of these tests. One notable result is that the quantity \( D\lambda\Omega \) obtained from static pull quadrature stiffness has a strong dependence on eccentricity ratio. Though damping can and does change with eccentricity in both externally pressurized and self-acting bearings, the magnitude of the observed change in \( D\lambda\Omega \) suggests \( \lambda \) is also changing. If this is the case, hydrostatic bearings may be shown to be more stable at lower rather than higher eccentricities, contradicting the rules for conventional bearings. Non-synchronous perturbation studies designed to control for eccentricity are required to determine the eccentricity-dependence of \( D \) and \( \lambda \) separately.

One notes from Figure 4 that damping (the slope of the QDS curve) is apparently affected by the frequency and/or amplitude of the vibration. This is consistent with a velocity-dependence in damping for a compressible fluid film with moving boundaries. Alternately, one may describe the phase of the “damping” response as beginning to lag. If it lags by more than 90°, the “damping” picks up a negative component, and pneumatic instability results. The authors find suggestions of this mechanism [5,9,11-13] but no completely satisfactory model or stability criteria. This is the subject of ongoing investigation.

CONCLUSION

The Bently-Muszynska parameters describing a hydrostatic gas bearing were seen to be consistently and predictably tied to the operational parameters of the bearing, indicating that they are suitable for inclusion in rotor-bearing models utilizing hydrostatic gas bearings. Studies in which geometric rather than operational parameters are varied will be needed to establish an empirical link between bearing design and model coefficients. This preliminary work also suggests avenues of study into the two distinct stability issues encountered in gas bearings, namely whirl and pneumatic hammer.

The suitability of hydrostatic gas bearings for any given application will be determined in part by the rotor-bearing dynamic models based on either predictions or direct measurements of the bearing characteristics. The stability margins for the two types of instability associated with gas bearings must also be determined prior to installation of any full-scale industrial applications.

REFERENCES


