Passive vibration isolation using a Roberts linkage

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The article present an ultralow frequency passive vibration isolation device as part of the preisolation stage for the Australian International Gravitational Observatory (AIGO). This isolator is based on the Roberts linkage and simulates a very long radius conical pendulum to provide two-dimensional isolation. It is designed as the second horizontal isolation component of the preisolator and it will support the remaining four isolation stages of the AIGO vibration isolator chain. A description of the device together with a theoretical model and its measured isolation performance are presented. With the current setup, we obtained 32 dB of isolation at 1 Hz. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1583862]

I. INTRODUCTION

Since the first interferometric gravitational wave detectors were developed, the main difficulty in attaining high strain sensitivity has been in reducing the various technical noise sources in the interferometer. The seismic motion of the test masses in such an interferometric antenna is a noise source that requires attention. Efforts to improve isolation against seismic motion have resulted in several proposed very low frequency passive isolators, such as the *X*-pendulum,[1] the folded pendulum (Watt's linkage),[2] and the Scott–Russel linkage.[3] Of these, the last two produced 90–100 dB isolation in a single stage device at ~10 and 2 Hz, respectively, although only the latter was capable of two-dimensional (2D) isolation.

Given that such exceptional results were obtained with a Scott–Russel based isolator[3] in our laboratory, it is worth noting the reasons for now trying the Roberts linkage (apart from exploring new ground). One main difference between the two linkages is the orientation of the supporting or fixing points (which have to be critically aligned with respect to the local gravitational field). For the Scott–Russel these points are vertically one above the other, while for the Roberts linkage these points are widely spaced in a horizontal plane. Thus in order to suspend the following stages centrally within the height of the preisolator, the Scott–Russel requires all of its 2D flexing joints to be designed as gimbals, whereas with the Roberts linkage this volume is naturally free, which allows simpler engineering. In addition it is foreseen that active tilt control may eventually be required and it seems easier to apply this as say a piezo actuator under each support point to keep the horizontal plane level than to apply it to the vertical case where the actual center is unavailable (due to centrally suspended following stages).

The Roberts linkage[4] is found in the literature as a structure that achieves near straight-line motion from rigid links and pivots and as such it provides another means to simulate the motion of a very long radius pendulum in a relatively short height in a similar manner to the previously mentioned linkages. As with the Scott–Russel, it may also be readily generalized to cylindrical symmetry to give *x*–*y* motion and was proposed for ultraflow frequency perisolation by Winterflood.[5] While its application as a soft suspension is apparently not well known, the structure has been employed previously for horizontal isolation inside an optical table leg.[6] However, the fact that the structure is actually a Roberts linkage does not seem to have been recognized.

We have built and used two small ultralow frequency seismometer devices using this structure,[7] and have now constructed a large model capable of suspending several hundred kilograms as part of an ultralow frequency preisolation structure intended for the Australian International Gravitational Observatory's (AIGO's) vibration isolation.

In this article we present performance measurements on this Roberts linkage isolator. We have designed the device to be the second horizontal element of two cascaded stages of very low frequency three-dimensional (3D) preisolation.[8] The 3D preisolator stage (consisting of inverse pendulum horizontal isolation and linear LaCoste vertical isolation) has both horizontal and vertical resonance frequencies below 100 mHz and is tilt rigid, allowing this Roberts linkage second stage to be cascaded from it and maintained in close alignment with the local gravitational field. The limitations of such passive structures due to ground tilt cross coupling to horizontal motion at the suspension point has been well covered elsewhere.[9] Here we concentrate on the Roberts linkage while in a forthcoming article we will present the entire cascaded preisolator.

The purpose of a preisolator is to reduce the residual seismic motion to well below the amplitude of the ocean wave driven microseismic peak. In conventional vibration isolators the worst residual motion normally occurs near 0.5–1 Hz due to the normal mode peaks of multiple pendulum stages. The residual motion causes many difficulties in optical cavity lock acquisition since the motion usually exceeds many optical wavelengths. Preisolation structures can reduce the seismic drive to the pendulum modes, in principle allowing the residual motion to be reduced to the nanometer level.[9] In addition, a preisolator is a useful device to allow

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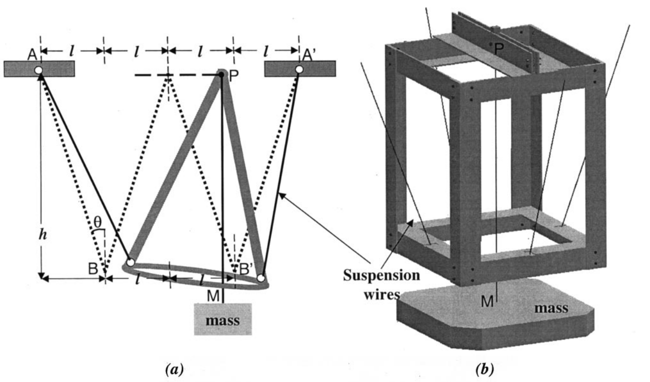

FIG. 1. The Roberts linkage diagram: (a) one-dimensional Roberts linkage with a suspended load from wire PM and (b) two-dimensional Roberts linkage with a suspended load.

smooth translation of the entire isolation chain together with the test mass by useful distances (e.g., millimeters). This can be used to set optical cavity length and to counteract any thermal expansion and earth tide effects.

In this article we will introduce the Roberts linkage geometry. Then we will present a theoretical model for the linkage and the transfer function setup. Finally, we present experimental results and compare them with the theoretical predictions.

II. THE DEVICE

As is the case with devices such as the Watt's linkage and Scott–Russel linkage,[5] the Roberts linkage preisolator illustrated in Fig. 1 works by causing the mass suspension point P to move in an almost flat horizontal plane, so that the suspended mass gravitational potential energy is almost independent of displacement. Small modifications in the geometry (such as by raising point P slightly above or below the plane of the suspension points) allow point P to move in a very shallow arc, which defines the effective length of an equivalent pendulum for the Roberts linkage.

One usually imagines this arc to form a very shallow potential well in which the suspended load moves, but in order to obtain very low frequencies it may need to be a significant potential hill (i.e., inverse pendulum) to cancel other restoring forces such as the flexing of the suspension wires, and the motion of the center of mass of the (ideally massless) rigid Roberts linkage frame. If a significant proportion of the restoring force is due to the suspended mass moving in such a potential well or hill, then the device's resonance frequency tuning become strongly dependent on the suspended load. A more useful arrangement is to attach fixed mass to the rigid frame well above point P to balance the mass of the structure below this point so that the center of mass of the rigid frame is at the same level as this point and at the level of the plane of the fixed wire suspension points. In this case the tuning becomes almost independent of the suspended load. The resonant frequency may then be adjusted by moving a small amount of mass up or down on the rigid frame, finely tuning the shallowness of the overall potential well.

A one-dimensional Roberts linkage with a suspended mass is shown in Fig. 1(a). It consists of a rigid frame attached to two wires. This linkage can be imagined as a *W* shape, where the sidelines represent the two suspension wires and the central inverted *V* is the rigid frame. If we take a copy of the one-dimensional linkage and rotate it 90° around a vertical axis with an appropriate rigid central section, a two-dimensional Roberts linkage[5] is obtained as shown in Fig. 1(b). This structure is overconstrained but intrinsic material flexibility and simple wire length adjustment make it a practical solution. The overconstraint can be removed by going to a three-wire design as we tested for the small seismometer,[7] but for installation in our cubic shaped preisolator, a square structure was greatly preferred.

The classical Roberts linkage[4] requires distances AB, BP, PB' and $A'B'$ to be equal and the distance AA' to be twice the distance BB' as shown in Fig. 1(a). The two-dimensional device illustrated is constructed of 60 mm×60 mm×60 mm aluminum angle with a height $h=692$ mm, from the upper to the lower suspension point. The distance l, as shown in Fig. 1(a), is equal to 273 mm. This gives a value of 21.5° of arc for the angle θ, between the vertical axis and the suspension wire of the linkage.

The four suspension wires are anchored to the structure at the bottom, and are supported by an adjustable threaded housing at the top, allowing the tension in the suspension wires to be matched and the linkage to be accurately positioned and leveled. The other stages of the suspension chain designed at AIGO[10] will be suspended from the Roberts linkage, approximately at the point P indicated in Fig. 1. There is extra weight on top of the frame, such that the position of the Roberts linkage center of mass is near the suspension point of the mass load (suspended mass in Fig. 1), to minimize the sensitivity of resonance frequency to the suspended load.

III. THEORETICAL MODEL

In this section we review the transfer function of a simple pendulum, and then generalize it to a physical pendulum.

The transfer function and the phase angle for a simple pendulum (see Fig. 2) have the following expressions:

$$\left|\frac{X_{\mathrm{output}}}{X_{\mathrm{input}}}\right|=\left[\frac{1+Q^{-2}}{(1-(\omega/\omega_{\mathrm{res}})^2)^2+Q^{-2}}\right]^{1/2}, \tag{1}$$

$$\alpha=\arctan\left[\frac{-(\omega/\omega_{\mathrm{res}})^2}{Q(1-(\omega/\omega_{\mathrm{res}})^2+Q^{-2})}\right], \tag{2}$$

where ω is the angular excitation frequency, ω_{res} is the angular resonance frequency, and the quality factor is given by $Q=1/\varphi_{\mathrm{Pend}}$. The pendulum loss has the following expression $\varphi_{\mathrm{Pend}}=\varphi(k_g+k_s)/k_s$, where φ is the material loss for suspension wire, k_g is the gravitational spring constant, and $k_s'=k_s(1+i\varphi)$ is the flexure spring constant. In addition, the spring constant of the pendulum (total spring constant) is given by $k_{\mathrm{Pend}}=k_g+k_s'=[k_g+k_s(1+i\varphi)]=(k_g+k_s)(1+i\varphi_{\mathrm{Pend}})=k(1+i\varphi_{\mathrm{Pend}})$, where k is the real part of the total spring constant.

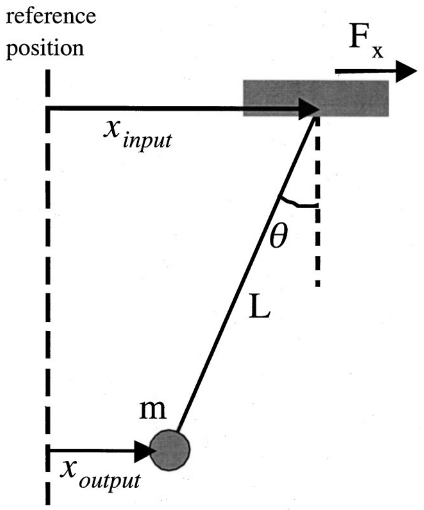

FIG. 2. Arbitrary simple pendulum.

Since the Roberts linkage utilizes a distributed mass, it is necessary to model it as an equivalent compound pendulum. For a physical pendulum (see Fig. 3) the transfer function at any point P on the line joining the suspension point with the center of mass and the phase angle have the following expressions:

$$\left|\frac{X_P}{X_O}\right| = \left[\frac{(1-\mathbf{A}(\omega/\omega_{\text{res}})^2)^2+Q^{-2}}{(1-(\omega/\omega_{\text{res}})^2)^2+Q^{-2}}\right]^{1/2}, \tag{3}$$

$$\alpha' = \arctan\left[\frac{-(1-\mathbf{A})(\omega/\omega_{\text{res}})^2}{Q((1-\mathbf{A}(\omega/\omega_{\text{res}})^2)(1-(\omega/\omega_{\text{res}})^2)+Q^{-2})}\right], \tag{4}$$

where

$$\mathbf{A} = 1-\frac{ml_{\text{CM}}l_{\text{OP}}}{I_O} = 1-\frac{ml_{\text{CM}}l_{\text{OP}}}{I_{\text{CM}}+ml_{\text{CM}}^2} \tag{5}$$

is a constant that depends on the suspension point moment of inertia (I_0), on the distance from the suspension point to the center of mass (l_{CM}), and to the measurement point $l_{\text{OP}} = l_{\text{CM}} + l_P$) and on the mass of the system (m).

It can be seen that for a simple pendulum the center of mass transfer function tends to zero at high frequencies, but for a compound pendulum, the transfer function tends to a finite value

$$\left|\frac{X_{\text{CM}}}{X_0}\right| \rightarrow \mathbf{A} = 1-\frac{ml_{\text{CM}}^2}{I_0} = 1-\frac{ml_{\text{CM}}^2}{I_{\text{CM}}+ml_{\text{CM}}^2}$$
$$= \frac{I_{\text{CM}}}{I_{\text{CM}}+ml_{\text{CM}}^2} = \frac{r_g^2}{r_g^2+l_{\text{CM}}^2}, \tag{6}$$

where r_g is the radius of gyration.

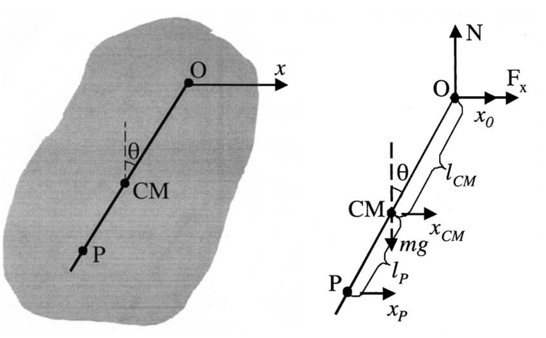

FIG. 3. Arbitrary physical pendulum.

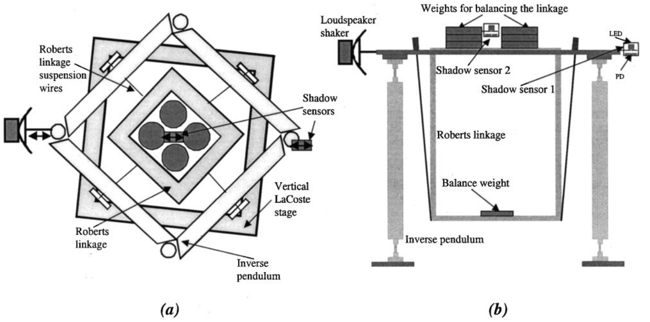

FIG. 4. Roberts linkage as part of the pre-isolator and measurement setup diagram: (a) top view of the preisolator; (b) lateral view; the vertical LaCoste stage is not shown as it is clamped to be inactive. A loudspeaker is used to shake the preisolator horizontally. The weights on top and bottom of the Roberts linkage are used to tune the resonance frequency of this device. The one-dimensional shadow sensors are placed as indicated and the measured transfer function is taken as the signal from shadow sensor (2) over the signal from shadow sensor (1). This drawing is not to scale.

If $\mathbf{A}=0$, the physical pendulum transfer function turns into the simple pendulum transfer function [Eq. (3)]. At this point, called the center of percussion

$$l_{\text{OP}(A=0)} = \frac{I_{\text{CM}}+ml_{\text{CM}}^2}{ml_{\text{CM}}} = l_{\text{CM}} + \frac{r_g^2}{l_{\text{CM}}}$$

and the compound pendulum behaves as a simple pendulum having this equivalent length.

IV. MEASUREMENT SETUP AND RESULTS

A schematic diagram of the experimental setup is presented in Fig. 4, where the position of the Roberts linkage in the preisolator is indicated. The device is attached to the first three-dimensional preisolator stage,[8] which consists of a horizontal inverse pendulum and a vertical LaCoste stage, which was clamped to be inactive for this measurement. Four high tensile steel wires of 1.4 mm diameter were used to suspend the Roberts linkage from the inverse pendulum. For these measurements the inverse pendulum was not optimally tuned but was used as tilt-rigid shake-testing frame. Its resonance frequency was 780 mHz. The Roberts linkage was carefully balanced and leveled, and tuned to achieve a resonance period of 20 s.

The tuning was achieved by distributing a mass of 60 kg on top of the Roberts linkage at a height of 140 mm above the suspension point level together with another 6 kg on the lower part of the linkage frame. The total mass of the system is such that the center of mass height was very close to the

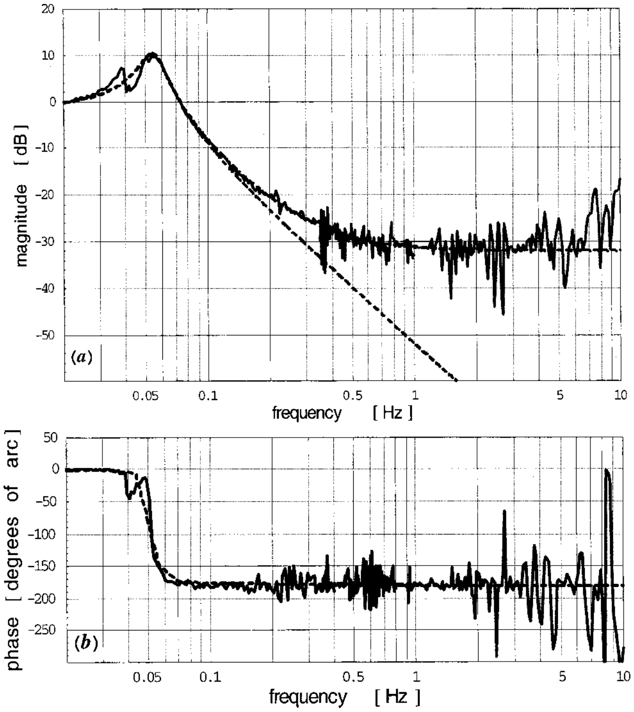

FIG. 5. (a) Roberts linkage transfer function. The two dashed lines represent physical pendulum model transfer functions for $\mathbf{A}=0$ (at the center of percussion) and for $\mathbf{A}=-0.03$, the last one fitting our experimental data. (b) Phase plots. The dashed line shows theoretical prediction (with $\mathbf{A}=-0.03$).

height of the Roberts linkage suspension points plane (this being what achieves the very low resonant frequency). A swept sine transfer function was recorded. To measure the transfer function shadow sensors were located as shown in Fig. 4, allowing measurement of the displacement of the first preisolator stage and the displacement of the Roberts linkage with respect to the ground.

The excitation signal was applied with a loudspeaker calibrated to provide a force of 6 N/A. This signal was applied at approximately 45° with respect to the two eigenmode directions of oscillation and the output was measured in the same direction. In order to keep a good signal to noise ratio, the measurement was made over three different frequency ranges with appropriate source amplitudes (2 V amplitude for 20–400 mHz, 1 V for 350 mHz–1 Hz, and 3 V for 1–10 Hz). For the experiments reported here differences between the periods of oscillation in the two orthogonal directions were encountered, such that one had 18.5 s period and the other had 25.5 s period. The reason for these differences can be assigned to small differences in distances between suspension points of the Roberts linkage in the two orthogonal directions as well as to imperfections in the suspension wire cylindricity and clamping isotropy. These last two factors could be improved by lapping the wires to match the collets as described in Ref. 11. Cross coupling between these two modes leads to a double peak response in the transfer function as is seen in Fig. 5. The main peak at 54 mHz shows the resonance frequency of the Roberts linkage in the measurement direction and the other small peak at 39 mHz is due to cross coupling with the perpendicular direction of oscillation. This arises because the excited direction is not exactly in line with a modal axis. The two dashed lines in Fig. 5(a) show theoretical transfer functions for the center of percussion $(\mathbf{A}=0)$ and for $\mathbf{A}=-0.03$. We also present the phase plots, for the measured data and for the physical pendulum model (dashed line) in Fig. 5(b). We used a value of 3.7 for the quality factor in this model.

Figure 5 shows the isolation of the Roberts linkage to be that expected for a physical pendulum when the constant $\mathbf{A}$ has a value of -0.03, yielding a maximum isolation of 32 dB. For use as a preisolator this is an adequate level of isolation.

Better isolation can be achieved for frequencies greater than 0.2 Hz if we suspend the isolation chain from the center of percussion of the Roberts linkage. However, suspending the load at the center of percussion will make the tuning and stability of the Roberts linkage dependent on loading and is therefore not desirable. An ideal design would achieve both low frequency tuning and high frequency isolation by placing both the center of percussion and the center of mass at the desired suspension point. In practice, this is impossible with rigid structure.

As normally occurs with low frequency structures, the Q factor is low because the negative spring created by the geometry cancels the real spring constant but leaves the dissipative terms unchanged.

V. DISCUSSION

A very low frequency horizontal vibration isolation device based on the Roberts linkage has been presented. The device has advantages in simplicity of design and economical usage of the space. The simplicity is also reflected in ease of setup. The device (only partially loaded) achieved a Q factor of ~4 with a resonance period of ~20 s. This suggests that with drift compensating feedback, the resonance period could be increased to 40 s (Q factor falls with resonance frequency squared for structural damping), particularly if the device were adequately loaded. With a resonance period of 18.5 s, an isolation of 32 dB at 1 Hz was obtained. This is sufficient to significantly attenuate the seismic excitation of the normal modes of an isolation chain. The 32 dB isolation floor is due to the distributed nature of the Roberts linkage mass, which prevents the center of percussion from being located at the center of mass. Better isolation could be achieved by reducing the mass of Roberts linkage frame compared to its mass load or by having the isolation chain attached to the center of percussion of the Roberts linkage. An isolation improvement of about 20 dB at 1 Hz should then be achieved. We point out, however, that suspending the following isolation stages from the center of percussion point makes the Roberts stage sensitive to the total mass load below it. Since it is impossible with a rigid structure to locate the center of percussion at the same point as the center of mass, a possible solution may be to add one or more load pendulum masses, which can change the center of mass (at ultralow resonance frequency) without affecting the high frequency dynamic response and the center of percussion.

Because of imperfections in construction the two orthogonal directions of oscillation of the Roberts linkage typically had 5 s difference in period. This shows up as a kink in the transfer function at 40 mHz, but has little effect above resonance. We chose to build a rectangular design for reasons of space and simplicity, but a three-wire device would reduce the number of factors contributing to this anisotropy and may have an advantage in some cases. Low frequency devices such as the Roberts linkage are always susceptible to drifts. Hence, in a working system we would always choose to control the working point with a very low frequency dc position control.

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